Baryon Acoustic Oscillations. Equation and physical interpretation

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***Abstract*—Baryon Acoustic Oscillations are a phenomenon occurred before matter-radiation decoupling, characterized because the baryonic matter perturbation presents oscillations, as the name suggests. These perturbations propagate like a pressure wave on the photon-baryon fluid produced by gravitational potentials, which join the baryonic matter, and collisions of baryons and photons, which scatter it. This paper shows the Baryon Acoustic Oscillations equation and it provides its physical meaning. Besides, it presents software CAMB as a tool to find BAO equation solutions and support for its physical description.**

***Key Word* —Acoustic oscillations, baryons, cosmological perturbations, Newtonian gauge.**

I. INTRODUCTION

First models about universe appeared few years after Albert Einstein published his General Relativity Theory. They described an isotropic and homogeneous expanding universe. However, since the universe contains inhomogeneities, which are the cause of great structures formation, it was necessary to develop a Cosmological Perturbation Theory. To be able to obtain a refinement when studying the structures formation, it is necessary to include baryonic matter. Baryon Acoustic Oscillations phenomenon is an approach to investigate baryonic matter behavior.

This paper will show the baryon acoustic oscillations equation and its physical interpretation, after a short discuss about cosmological perturbation theory to introduce the topic. Physical conditions of the universe that make baryon acoustic oscillations to exist are also explained.

For this purpose, it will be showed how baryon acoustic oscillations equation can be approximated to a harmonic oscillator equation. This fact, together with some approximate solutions will make easier to understand its physical interpretation. Software CAMB was used to complement the investigation of baryon acoustic oscillations phenomenon through analysis of simulations and graphics.

II. COSMOLOGICAL PERTURBATIONS

The universe must contain inhomogeneities that explain grouping of matter, like galaxies clusters (figure 1). These inhomogeneities occurs on an idealized universe, thus, it is convenient first to discuss about the ideal universe.

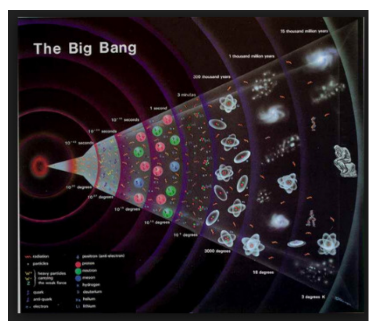

Figure 1. Inhomogeneities in the universe. Picture taken from page https://www.universetoday.com/54756/what-is-the-big-bang-theory/.

A. Ideal universe

The ideal universe is depicted by a couple of equations known together as *Friedmann Equations* [1]:

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3}, \qquad (1)$$

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$$\frac{a''}{a} = -\frac{4\pi G}{3}(\bar{\rho} + 3\bar{P}) + \frac{\Lambda}{3}, \quad (2)$$

where a is the *scale factor*, which only depends on *cosmological time*, t, Λ is the *cosmological constant,* $\bar{\rho}$ and $\bar{P}$ are the average density and pressure respectively. Symbol prime represents derivatives with respect to cosmological time. The Friedmann equations describe the evolution of a homogeneous and isotropic universe, which is in an increasing expansion.

From the terms involved in Friedmann equations it is possible to define some useful parameters like *Hubble rate* $H = \frac{a'}{a}$, which is a measurement of the universe expansion speed, *Hubble radius* $R_H = \frac{c}{H}$ is the distance for which universe expansion speed is higher than light speed, and *Hubble time* $t_H = H^{-1}$, is the lapse needed by light to travel a Hubble radius.

B. Definition of cosmological perturbations

As it was told before, the real universe must contain inhomogeneities that explain grouping of matter. These inhomogeneities are caused by perturbations in the universe. Because of, it is important to define the cosmological perturbations.

We count with two different space-times. One of them describes the ideal homogeneous and isotropic universe, ruled by Friedmann equations. The other one describes the real universe, which contains grouped matter. We can associate a point p of real space-time with a point of ideal space-time $\bar{p}$ via a mapping $p = \phi(\bar{p})$. This association is called a *gauge choice*. The mapping should be one to one, but it is not unique and making a different mapping to imply to do a *gauge transformation.* In this way we can study the behavior of the real universe through the information of ideal universe, which is already known enough. Now we can define the perturbations [2, 3, 4] as the difference of the quantities of real universe and ideal universe, like, for example, perturbations on density and temperature:

$$\delta\rho = \rho - \bar{\rho},\ \delta T = T - \bar{T}\ . \quad (3)$$

Also, we can define the fractional perturbations as following:

$$\delta_\rho = \frac{\delta\rho}{\bar{\rho}},\ \delta_T = \frac{\delta T}{\bar{T}}\ . \quad (4)$$

C. Temperature perturbations

The perturbations on the temperature can be observed now in the *CMB (Cosmic Microwave Background)* and they have an order of 10^{-5}. Since the average temperature of CMB is 3 K, it means that temperature perturbations are of the order of some μK . The temperature pertubations in CMB were studied by the first time at 1995 by *COBE (Cosmic Background Explorer)* satellite [5], but its observations has been improved by further project, being *PLANCK* the last one, in 2013 [6].

D. Perturbations on metric

Perturbations in space-time are described through the metric:

$$g_{\mu\nu} = \begin{pmatrix} -(1+2\Psi) & 0 \\ 0 & a^2(1+2\Phi)\delta_{ij} \end{pmatrix}. \quad (5)$$

This metric is known as *Newtonian gauge* [7]. It includes only scalar perturbations where Ψ and Φ are scalar functions which depend on space-time coordinates, with the conditions $|\Psi| \ll 1$, $|\Phi| \ll 1$. The newtonian gauge metric allows to describe the formation of great structures, where Ψ and Φ play the function of perturbed gravitational potencials.

III. BARYON ACOUSTIC OSCILATIONS

The *baryon acoustic oscillations* (***BAO*** from now on) is a phenomenon ocurred at the early times of universe, before the decoupling of matter and radiation, where the perturbation of baryonic matter propagated as a wave. This section shows the equation which rules BAO and the physical meaning of it. However, it is convenient, in a first place, to indicate the phsysical conditions of the universe at age of interest, which made posible the presence of BAO.

A. Physical conditions of the universe before matter-radiation decoupling

Time of decoupling is placed at 300.000 years after *Big-Bang*. Before that, the universe temperature was around $3000\ K$ making the photons to be very energetic. Photons interacted with baryons, which, in the cosmology context, refer not only protons but also electrons, via *Compton scattering*. This strong interaction is known as *tight coupled limit* and it caused that photons and baryons moved together like an unique fluid, the *photon-baryon fluid*. Because of, the model to describe this behavior is known as *fluid approximation*. Energetic photons were able to scatter baryons and they avoid that proton and electrons to join into neutral atoms [8].

The previous described conditions finished when the Hubble rate becomes higher than scatter rate. Photons are not energetic enough to scatter baryons, occurring the decoupling of matter and radiation Protons and electrons to join in neutral atoms and photons follow free ways, being possible to be observed currently in the CMB.

B. The equation of baryon acoustic oscillations

The equation that rules the BAO phenomenon is:

$$\ddot{\delta}_b + \frac{\dot{R}}{1+R}\dot{\delta}_b + k^2 c_s^2 \delta_b = -k^2\Psi - 3\frac{\dot{R}}{1+R}\dot{\Phi} - 3\ddot{\Phi}\,, \tag{6}$$

where overdots represent derivatives with respect to *conformal time* η, which is written in terms of cosmological time as:

$$\eta = \int_0^t \frac{dt'}{a(t')}\,. \tag{7}$$

Besides, R is the *photon-baryon* ratio:

$$R = \frac{3\bar{\rho}_b}{\bar{\rho}_\gamma} \tag{8}$$

and c_s is the *sound speed,* defined via photon-baryon ratio as:

$$c_s = \sqrt{\frac{1}{3(1+R)}}\,. \tag{9}$$

Thus, (6) is a second order linear differential equation for baryon fractional perturbation δ_b. It is written in terms of wave number k, since it has applied *Fourier transform* to it, indicating that BAO are analysed in term of lenght scales instead of spatial coordinates.

C. Physical meaning of bao equation

To show the physical meaning of BAO equation (6) we can rewritte it in a schematic way as:

$$\ddot{\delta}_b + b\dot{\delta}_b + \omega^2\delta_b = F\,. \tag{10}$$

This is a much more familiar equation. It is a *damped forced harmonic oscillator* equation, where the forced term is given by gravitational potentials. It is not strange to obtain this kind of equation if we consider that BAO phenomenon is produced by a competition between pressure forces, produced by Compton scattering, which separate the baryonic matter, and gravitational potentials, which group it (view figure 2 for a descriptive picture).

Figure 2. Pressure on baryons in a gravitational potential well. This picture has been modified from page http://background.uchicago.edu/~whu/power/bao.html.

It creates oscillations which propagate on the photon-baryon fluid (figure 3).

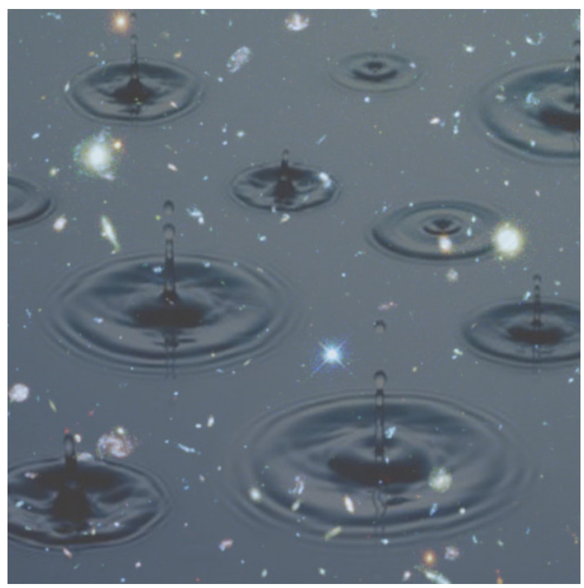

Figure 3. Baryonic matter oscillates on the photon-baryon fluid. This picture has been taken from page http://zuserver2.star.ucl.ac.uk/~mts/webpage_dev/BAOs.html.

The kind of movement for baryonic matter, described above, is analogue to the propagation of a gas pressure wave, just like sound waves on the air. It justifies the term *acoustic* for BAO phenomenon, and others derived, like sound speed, etc.

1. The oscillations

The baryonic matter perturbation propagates in an oscillatory movement [8] given by:

$$\delta_b = \delta_{b_{ini}} \cos(k c_s \eta) \; . \tag{11}$$

It indicates that the propagation follows a *cosine* form around a perturbation $\delta_{b_{ini}}$, provided by initial conditions.

The movement of baryonic matter, analogue to a wave, suggests to define a distance reached by the propagation, known as *sound horizon*:

$$d_s = \int_{\eta_{ini}}^{\eta} c_s d\eta' \; . \tag{12}$$

The propagations finishes when matter-radiation decoupling to occur, since decoupling is the end of the conditions to allow BAO. When decoupling to present, the baryonic matter perturbations reach a maximum sound horizon and they are frozen in the CMB. After that, they keep moving, following the universe expansion. What we can see today is a kind of spherical shells, where the radius of the spheres, the maximum sound horizon, is 150 Mpc, and shells are 30 Mpc thick [9] (figure 4).

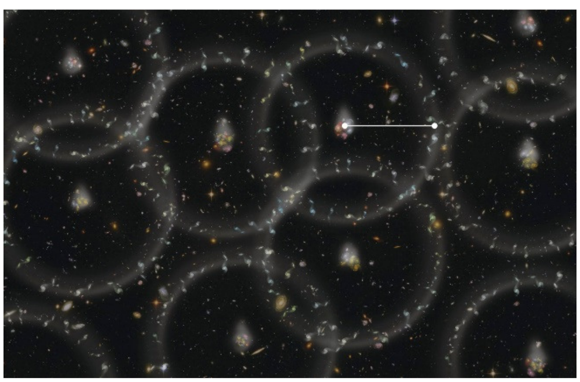

Figure 4. Illustration of BAO. Picture taken from page http://apod.nasa.gov/apod/ap140120.html.

2. Oscillations damping

BAO present a damping [1] given by:

$$\delta_b \sim e^{\frac{k^2}{k_D^2}} \; , \tag{13}$$

with

$$k_D = \frac{2\pi}{\lambda_D} \; . \tag{14}$$

The *damping wavelenght* λ_D is the average lenght traveled by a photon in a Hubble time. It indicates that baryon perturbations are vanished by the interaction with photons, for scales lower than λ_D.

IV. GRAPHICS AND SIMULATIONS

There are annalytic solutions for (6), for which (11) and (13) are aproximated results. You can see, for example [1, 10]. However, in this paper, is preferred to follow a different way, which consists to present graphics of solutions, since it is considered that it is a better way to show the physical meaning of BAO. For this purpose, it is gotten support from the software **CAMB**: **Code for Anisotropies in the Microwave Background** [11]. This software was created by Antony Lewis and Anthony Challinor at 2003. The software has been continously improved and last version was released at August 2017 (keeping in mind the date when this paper was written).

The software provides, among others, solutions of baryonic matter perturbations against wavenumber k. Because of, it is necessary to set a time, or, what is equivalent, to set a *redshift*. For following graphics it is set a redshift $z = 1100$, since it belongs to the matter-radiation decoupling redshift, which is time when BAO just to finish.

In the figure 5 and followings, it is used GNUPLOT tool [12] to graph δ_b vs k from data provided by CAMB. In the figure 5 can be seen some of the behaviors already mentioned, like oscillations and damping.

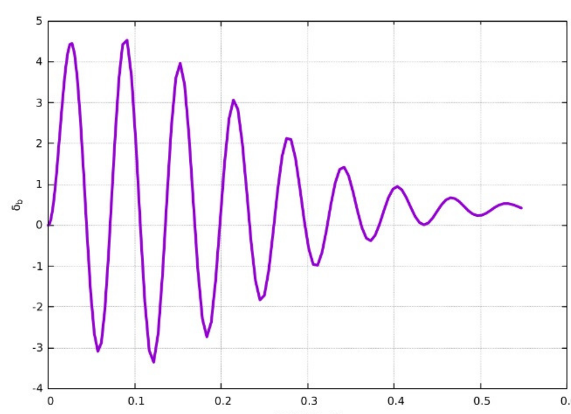

Figure 5. Baryonic matter perturbation.

If graphing δ_b vs $\log(k)$ (figure 6), it can be noted that the oscillations are not present at high scales. This is because δ_b starts to oscillate only for scales inside the Hubble radius where there is a causal interaction between photons and baryons, which produces the pressure to lead baryonic matter to oscillate.

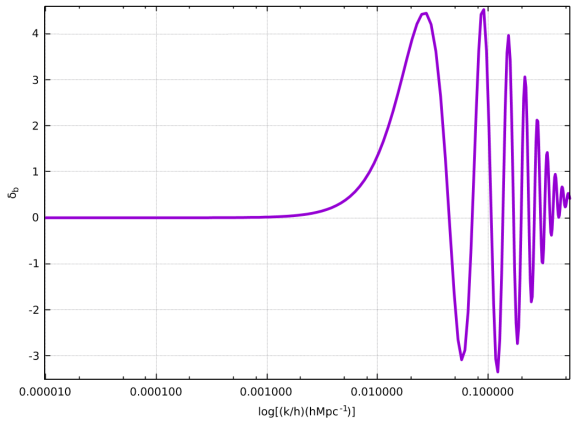

Figure 6. Baryonic matter perturbation, logarithmic scale.

Graphing $\frac{\delta_b}{k^2}$ vs $\log(k)$ (figure 7), it is observed that baryonic matter perturbation value is not 0 for scales lower than Hubble radius, but there is an overdensity that depends on initial conditions.

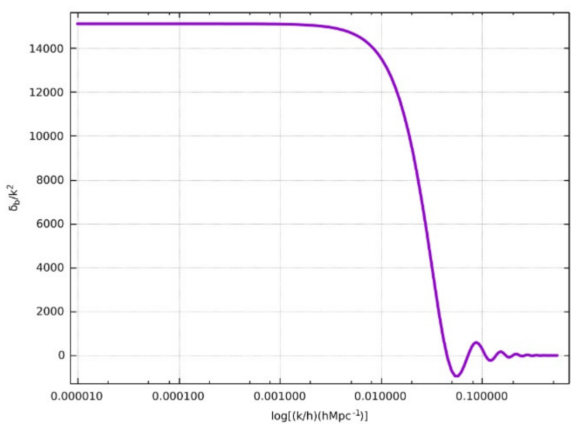

Figure 7. Baryonic matter perturbation/k2

Now, graphing δ_b^2 vs k (figure 8), it is clear that odd peaks are higher than even peaks [1]. It means that overdensities are higher, in magnitude, than underdensities. To explain that, consider that, when there are underdensities, pressure decreases, making more difficult for baryons to escape from potential wells. Baryonic matter density can´t continue lowing and it will return to be overdense. It also explains that, at the end, baryonic matter agroupations to prevail, influencing great structures formation. The previously explained behavior is interesting enough to be highlighted and it is very specific of BAO physics involved.

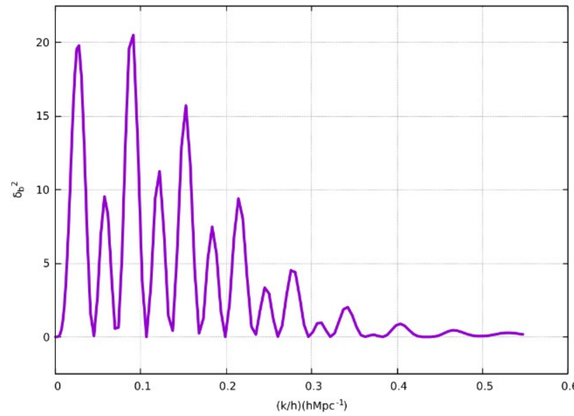

Figure 8. Baryonic matter perturbation raised to square.

Studies about great structures formation normally have in account dark matter since it represents around 30% of total content of matter in the universe, while baryonic matter only represents around 4% [1]. It means that the investigations about baryonic matter, including BAO phenomenon, provide a refinement of this topic and can explain how galaxies in the universe were formed (figure 9).

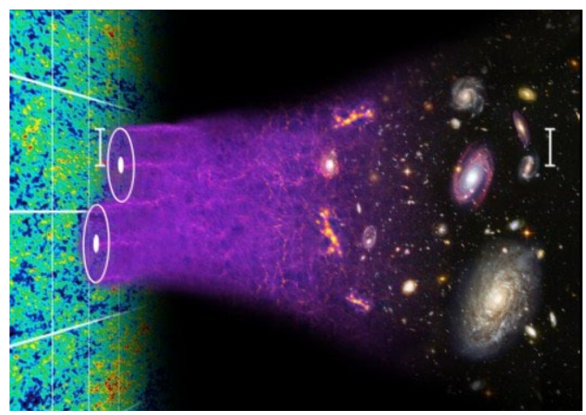
Figure 9. Baryon acoustic oscillations help to explain great structure formation. This picture has been taken from page http://www.sdss3.org/surveys/boss.php.

V. CONCLUSIONS

Once finished this discussion, it is possible to provide main conclusions:

- BAO equation can be considered, in a simplified way, as a a damped harmonic oscillator forced since it presents all of its features, being that BAO are the result of a competition between gravitational potentials, which group the baryions, and Compton scattering between baryons and photons which causes a pressure that disipates the baryonic matter.

- Thus, baryonic matter propagates as a pressure wave on the photon-baryon fluid with a speed given by (9), emulating the propagation of a sound wave, being this what justifies the name of baryon *acoustic* oscillations
- Oscillations propaging at sound speed define a distance called sound horizon. After matter-radiation decoupling, BAO finished and they are frozen. What we can see today are a kind of spheric shells. Sphere radius is 150 Mpc and shell thick is 30 Mpc. These baryon grouping are key to analyse great structures formation, because it represents a fine tuning, since the baryonic matter contribution to structures formation is included.
- Through CAMB software it was possible to complement the analysis of the physical behavior of BAO. Software simulations showed BAO features like damping, behavior for superhorizon scales and how the magnitude of overdensities is higher than the one of underdensities.
- The fact that overdensities to prevale indicates that baryon matter agroupations should have in account to model great structures formation, consituting a refinement of these investigations.

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