Characterization of the radiation and matter transfer functions

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***Abstract* —This paper shows the characterization of the matter and radiation transfer functions. The transfer functions provide information about behavior of every species perturbation during radiation epoch, which allows to determine the evolution of these species during matter epoch. Graphics of the radiation and matter transfer functions, using the software CAMB are presented, in order to explain how large structures were formed.**

***Key Word* — Cosmological perturbations, Newtonian gauge, transfer function.**

I. INTRODUCTION

The transfer function is a mathematical tool to explain a physical phenomenon in the period between inflation and Cosmic Microwave Background (CMB) [1]. It allows to explain large galaxy clusters formation. The evolution of Universe is showed in the Figure 1.

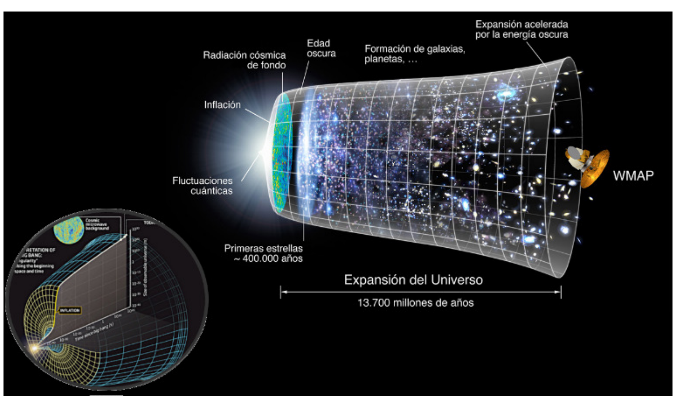

Figure 1. Evolution of Universe. Taken from web site NASA [2]

At bottom of Figure 1, the period of the transfer function explaining physical phenomena is shown. This period is called radiation epoch [3]. This paper focuses on this epoch, so, the solutions of the equations are presented in this period.

The transfer function explains information transfer of the perturbations behavior at radiation epoch, in order to understand the perturbation evolution at matter epoch and how the perturbations grew up to become large structures.

This paper shows the radiation and matter transfer function equations, related graphics and physical information about evolutions of perturbations.

II. CONTENTS

To obtain and to study the radiation and matter transfer functions, it is necessary to start from the first model that explains the evolution of a homogenous and isotropic universe, also known as the background universe. Then, the perturbations theory is presented, allowing to get a new model to explain the dynamic of the perturbed universe, which is a better approximation to explain the current observations of the universe. Perturbation theory will be the methodology to find the equations of radiation and matter transfer functions. These functions will be graphed using the software CAMB and the physics that each of these involves will be discussed.

A. The Background Universe

Friedmann, Lemaitre, Robertson and Walker set a method to explain the Universe. This is a metric which describes a homogeneous, isotropic, expanding and flat universe [4,5]. It is called *FLRW metric*. Written in terms of conformal time (τ) is:

$$ds^2 = a^2(\tau)\left(-d\tau^2 + \delta_{ij}dx^i dx^j\right) , \qquad (1)$$

where a is the scale factor. Using FLRW metric, Einstein field equation can be solved. Einstein field equation is represented as:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + g_{\mu\nu}\Lambda , \qquad (2)$$

where $G_{\mu\nu}$ is the Einstein tensor, which describes the universe geometry. G is the gravitational constant; $T_{\mu\nu}$ is the Stress-Energy tensor. Regarding FLWR metric, $T_{\mu\nu}$ corresponds a

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perfect fluid model. $g_{\mu\nu}$ is the metric tensor (FLRW), and Λ is the cosmological constant [5].

Friedman wanted to describe the universe dynamics. By solving the equation (2) with the metric (1), *Friedmann equations* are obtained [1,3,6]. These equations are:

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi G}{3}a^2\rho + \frac{a^2\Lambda}{3} \quad , \tag{3}$$

$$2\left(\frac{a'}{a}\right)' + \left(\frac{a'}{a}\right)^2 = 8\pi G a^2 P + a^2\Lambda . \tag{4}$$

From where can defined the Hubble parameter as $H = \frac{a'}{a}$, which refers to universe rate expansion, and the Hubble radius which is the limit of causal interactions. It is $R_H = \frac{c}{H}$, where c is the light speed [5,7].

B. The Perturbed Universe

The universe described by FLRW couldn't explain the new observations of the universe, made with different probes. One of the observations was the Cosmic Microwave Background (CMB), as shown in the Figure 2. The observations of inhomogeneities in the universe are presented. Because of, it was necessary to include the perturbations theory.

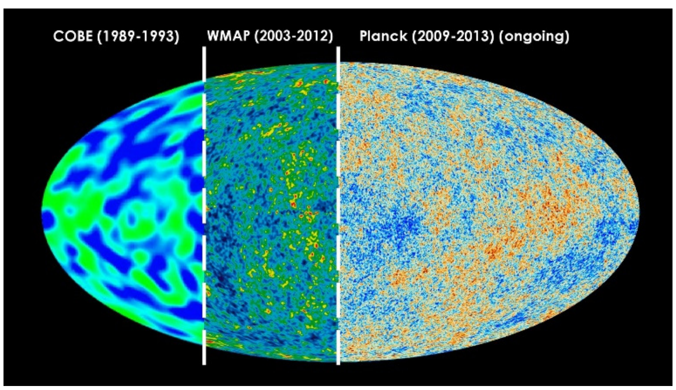

Figure 2. CMB reported by missions COBE, WMAP and Planck. Taken from web site NASA [8]

The perturbation theory proposes to represent the perturbed universe using the background universe. However, to make the identification between both, background and perturbed universes, a gauge must be chosen [5,9,10]. For this research the Newtonian gauge is selected. It is because this gauge explains the structures formation [9,11]. This gauge introduces two perturbed potentials, ψ y ϕ, in the FLRW metric, obtaining a perturbed metric described by:

$$ds^2 = a^2\left(-\left(1+2\psi\right)d\tau^2 + \left(1+2\phi\right)\delta_{ij}dx^i dx^j\right). \tag{5}$$

Other perturbed magnitudes are defined. They are the density perturbation and the temperature [11,12] given by:

$$\delta = \frac{\delta\rho}{\rho} \quad \text{y} \quad \theta = \frac{\delta T}{T} \quad . \tag{6}$$

With equation (5), Einstein field equation (2) can be solved to obtain:

$$k^2\phi + 3H\left(\psi' + H\phi\right) = -4\pi G a^2 \delta\rho \tag{7}$$

$$\psi'' + \phi\left(2H' + H^2\right) + H\left(2\psi' + \phi'\right) - \frac{2}{3}k^2\left(\phi - \psi\right) = 4\pi G a^2 \delta P \ . \tag{8}$$

Other equation to be solved is the Boltzmann equation:

$$\frac{df}{dt} = C \quad , \tag{9}$$

where f represents the distribution function for the species and C is the collision term [1,12]. This equation represents the evolution of the distribution function of species, taking into account the interaction between them.

In this investigation, the Boltzmann equation for photons and for dark matter is calculated. Dark matter is chosen because it is more abundant than baryonic matter [13]. The Boltzmann equation for photons is given by the equation:

$$\theta' + ik\mu\theta - \phi' + ik\mu\psi = -\chi'(\theta_0 - \theta + \mu v_b) , \tag{10}$$

where k is the wave number, which indicates that we are working in the Fourier space, θ is the perturbation in the temperature. μ is a parameter that indicates the direction of the photons. v_b is the peculiar speed of the barons. θ_0 it is the temperature monopole and χ is the optical depth, which determines how opaque is the universe, at a determined time [12].

The Boltzmann equations for matter are:

$$\delta' + ikv_{cdm} - 3\phi' = 0 \quad , \tag{11}$$

$$v'_{cdm} + \frac{a'}{a}v_{cdm} + ik\psi = 0 \quad . \tag{12}$$

Equation (11) determines the evolution of dark matter density, where v_{cdm} is the peculiar velocity of dark matter [12].

Equation (12) represents the evolution of the peculiar speed of dark matter.

With the equations (7), (8), (10), (11) and (12), the system of equations that describes the dynamics of the perturbed universe is complete. The solutions of the system equations are the radiation and matter transfer functions [12].

C. Radiation Transfer Function.

In order to obtain the radiation transfer function, equation (10) is rewritten as:

$$\theta'+(ik\mu-\chi')\theta=\phi'-ik\mu\psi-\chi'(\theta_0+i\mu v_b) \quad (13)$$

This is a differential equation for the temperature perturbation θ [12,14]. The equation (13) is solved by *line of sight* method [15], which allows to express the solution in two functions: a geometric one, given by the spherical Bessel functions (j_l) and other for interactions, S, known as the temperature sources function [12,15]. This sources function is compound by three effects known as the Sachs Wolfe effect, Doppler Effect and the Sachs Wolfe Integrated Effect [12]. The solution of it is the radiation transfer function and is given by:

$$\theta_l(\tau_0,k)\,\square\,[\theta_l(\tau_0,k)+\psi_l(\tau_{dec},k)]\,j_l\left(k\left(\tau_0-\tau_{dec}\right)\right)$$
$$+k^{-1}v_b(\tau_{dec},k)\,j_l{}'\left(k\left(\tau_0-\tau_{dec}\right)\right)$$
$$\int_{\tau_{dec}}^{\tau_0} d\tau[\phi_l{}'(\tau_0,k)+\psi_l{}'(\tau_{dec},k)]\,j_l\left(k\left(\tau_0-\tau_{dec}\right)\right) \quad (14)$$

In the first line of equation (14), the Sachs Wolfe effect is shown; this effect describes the temperature perturbation evolution and its interaction with the gravitational potential. In the second line the Doppler effect can be observed. This term describes how the peculiar speed of the barons affected the distribution of the photons. Finally, line three belongs to the integrated Sachs Wolfe effect, which is an integral that describes the evolution of the gravitational potentials from the time of decoupling to our days.

Thus the radiation transfer function shows the photons evolution during the radiation epoch, which makes possible large structures to be formed. Using the Software *Code for Anisotropies in the Microwave Background* (CAMB) [16], the radiation transfer function is plotted, obtaining figure 3.

In the figure 3, the temperature perturbation evolution with respect to the scale can be observed. For super-horizon scales, a constant behavior is observed. This is because for these scales the photons do not interact causally with other particles. However, for sub-horizon scales, an oscillatory behavior is observed. This is due to the photons to interact with the baryons for these scales, constituting the baryonic acoustic oscillations (BAO) phenomenon [12].

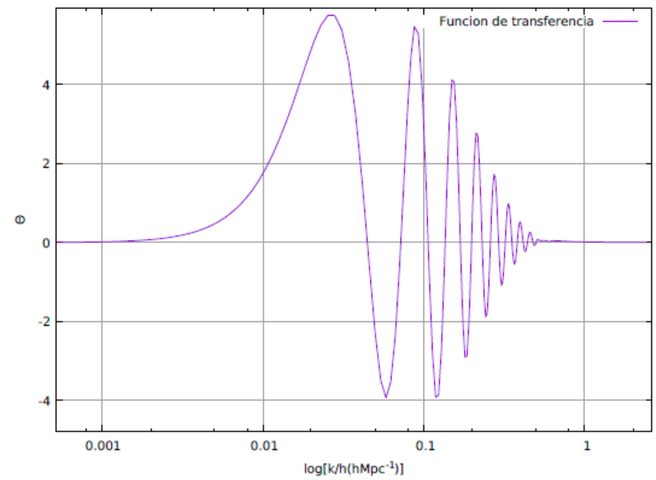

Figure 3. Radiation transfer function obtained with CAMB. This graphics represents the evolution of temperature perturbation with respect to the scales.

This oscillating behavior also allows to deduce that matter and radiation were grouped and separated, since the oscillations represent overdensities and subdensities [12]. The presence of overdensities imply that matter grouped, which later caused the large structures formation.

D. Matter Transfer Function.

Being interested to know matter perturbations evolution and being able to know at what moment the large structures were formed, we proceed to find the matter transfer function.

To find this function we start from equations (11) and (12). Combining them we obtain the *dark matter perturbations master equation:*

$$\delta_{cdm}''+H\delta_{cdm}'=k^2\phi+3H\phi'+3\phi'' \quad (15)$$

This is a differential equation that describes the evolution of dark matter perturbation δ_{cdm} [12,14]. Through the software CAMB, the equation (15) is solved to obtain the graph of Figure 4.

In Figure 4, we can observe the matter transfer function for different epochs of evolution. For all periods a constant behavior is observed, for super horizon scales. However, for sub-horizon scales it is possible to see a growth in the perturbations.

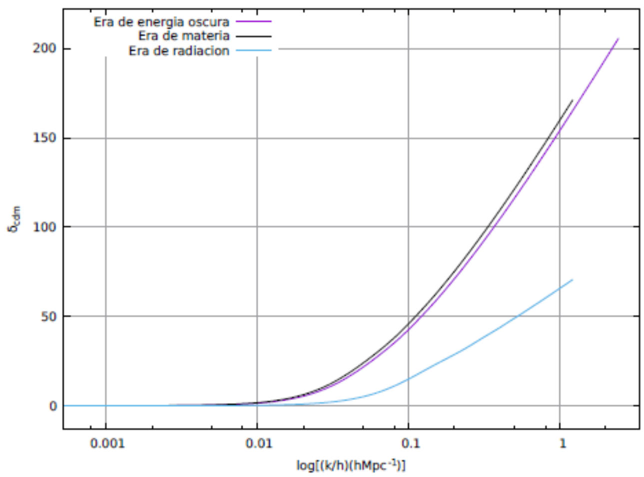

Figure 4. Matter transfer function obtained by CAMB. These graphics represent the evolution of cold dark matter perturbation with respect to the scales, at different periods.

At radiation epoch, represented by the blue line, a growth in perturbations is observed, although it is not significant with respect to the other epochs. The black line represents the perturbations evolution at matter epoch. This growth is significant, indicating that in this epoch the great structures were formed. This affirmation is based on the fact that during this time the perturbations grew, which means that the groupings of matter were abundant, being the beginning of the structures formation that we observe today. The purple line corresponds to the perturbations evolution at cosmological constant epoch. It is observed that, even with the expansion, the perturbations continue increasing, therefore it is expected that in future large structure formation will continue.

III. CONCLUSIONS

Obtaining transfer functions to require, as initial steps, the cosmological perturbation theory and the Boltzmann equations. Using the Newtonian gauge, we get a set of equations that relate the gravitational potentials with the density and pressure perturbations. These equations are known as perturbed field equations and correspond to a first approximation to know the cosmological perturbation evolution, since, through them, there are solutions for gravitational potentials.

By using the Boltzmann equations, the photons perturbation equation (13) is obtained. This equation is solved through a method known as line of sight, from where we have a temperature perturbation model for today.

Equation (14) shows three terms. These terms are known as the Sachs-Wolfe effect, Doppler effect and the integrated Sachs-Wolfe effect. The Sachs-Wolfe effect describes the monopole of temperature evolution and the gravitational potential. The Doppler effect contains information of the baryons speed. The SWI effect shows the evolution of gravitational potentials. Together, these effects to constitute the characterization of the temperature transfer function.

The matter perturbation evolution is determined by equation (15), known as the dark matter master equation.

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