Implementation of a new matheuristic to solve the vehicle routing problem with simultaneous deliveries and pick-ups – VRPSPD

Implementación de una nueva matheuristica para resolver el problema de ruteo de vehículos con entregas y recogidas simultáneos - VRPSPD

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DOI: https://doi.org/10.22517/23447214.23791
Scientific and technological research article

Abstract— The objective of this article is to present a new methodology for solving the Homogeneous Vehicle Routing Problem with Simultaneous Pickups and Deliveries (VRPSPD). The methodology integrates a matheuristic using the Chu-Beasley genetic algorithm and mixed integer linear programming, based on the Branch-and-Bound procedure. The best configuration obtained from the genetic algorithm is improved through the use of constructive heuristic methods in determining sub-problems, which contribute to the creation of the initial population necessary in the stage of local improvement. The goal is to determine minimum-cost routes that satisfy the pick-up and delivery demands of a group of geographically dispersed customers, while considering the restrictions of the system and the number of vehicles required. The methodology is implemented in C++ and a solver CPLEX software is used to find the solution. Results from test instances in specialized literature demonstrate the efficiency of this new hybrid model, showing good results in short computation times. This article is based on the author's doctoral thesis in engineering and presents similar topics discussed in international congresses, which may explain the similarities in the consulted bibliographical references.

Index Terms—Genetic algorithm of Chu–Beasley, heuristics, optimization, pick-up and delivery, vehicles routing.

Resumen— El objetivo de este artículo es presentar una nueva metodología para resolver el Problema de Encaminamiento de Vehículos Homogéneos con Recogidas y Entregas Simultáneas (VRPSPD). La metodología integra una matemática que utiliza el algoritmo genético Chu-Beasley y la programación lineal entera mixta, basada en el procedimiento Branch-and-Bound. La mejor configuración obtenida a partir del algoritmo genético se mejora mediante el uso de métodos heurísticos constructivos en la determinación de subproblemas, que contribuyen a la creación de la población inicial necesaria en la etapa de mejora local. El objetivo es determinar rutas de coste mínimo que satisfagan las demandas de recogida y entrega de un grupo de clientes dispersos geográficamente, teniendo en cuenta las restricciones del sistema y el número de vehículos necesarios. La metodología se implementa en C++ y se utiliza un software solver CPLEX para encontrar la solución. Los resultados de instancias de prueba en literatura especializada demuestran la eficiencia de este nuevo modelo híbrido, mostrando buenos resultados en tiempos de computación cortos. Este artículo está basado en la tesis doctoral en ingeniería del autor y presenta temas similares discutidos en congresos internacionales, lo que puede explicar las similitudes en las referencias bibliográficas consultadas.

Palabras claves— Algoritmo genético de Chu – Beasley, entregas y recogidas, heurísticas, optimización, ruteo de vehículos.

This manuscript was sent on December 5, 2021 and accepted on June 24, 2022.
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THE content of the research on the vehicle routing problem -VRP comprises an important series of events that include very brief and simple cases to situations that, due to their complication and complexity, deserve a greater level of depth in the scientific context.

It is well known to all that the practical solution of this class of routing problems focuses on determining a viable set of routes served by a set of vehicles to satisfy the delivery and pick-up service requested by customers, at a reasonable cost of transport, which is caused in the development of the distribution processes of supply of products, people and merchandise.

Structurally, this routing problem aims to find a set of solutions that can be good or optimal, achieved with the use of heuristic, metaheuristic or matheuristics techniques that are normally affected by restrictions associated with the number of vehicles used, their capacity, the location of pick-up and delivery sites, preparation times, route duration times, use of a single depot or multiple depots, topography of the land, etc.
All routes begin and end at the depot. The load of the vehicles is made up of the products or merchandise that must be delivered and what must be collected simultaneously according to the services requested by the clients. It is convenient to specify that in this case combinatorial optimization is applied, where there are many variables accompanied by a good number of parameters and in addition, the different versions of the vehicle routing problem are of NP-Hard class because polynomial time is not used in the solution. One of the pioneers in this kind of routing problem was H. Min [1], who devised a three-stage constructive heuristic to solve the case of a distribution system for the public library in Franklin County, Ohio. His solution was reflected in saving time and distance using a mathematical model.

Below is a list of the most relevant variants of this routing problem from 1989 to 2018:

- Group 2: is made up of the “one-to-many-to-one” group of problems. Its analog is the route generation VPR when mixed vehicles are used – MVRP. In this case, the customer requires one of two services: delivery or pick-up.

- Group 3: is made up of “one by one” VPRs where each product is treated as a requirement that comes from an origin and has a defined destination. Examples of this group are courier operations and the provision of door-to-door transport.

In addition, it is convenient to take into account that the use of exact techniques in the solution of vehicle routing problems with simultaneous deliveries and pick-ups - VRPSPD has presented difficulties related to the use of a considerable number of variables and restrictions [24][25].

Cases like these are easily solved with the use of the matheuristics proposed in this article, which allows finding good solutions, usually close to the optimal solution in relatively short computation times.

It has been shown that most of the applications of the VRPSPD are presented in the different processes of direct and reverse logistics, because the organizations in their mission must carry out operations and activities related to the direct and reverse flow applied for the distribution of finished products, semi-finished products and raw materials. The activities carried out in vehicle assembly plants, soft drink, beer, or soft drink bottling plants are cited as examples. In the latter, they occur when visits to customers are scheduled, full bottles are delivered to them and empty bottles are collected, simultaneously fulfilling deliveries and pick-ups, satisfactorily meeting customer requirements by applying the optimal route with minimum costs in the supply chain.

Another example of the VRPSPD is illustrated in: the transport of passengers either in the transfer or in the collection in different places; in home services, where goods and documents are delivered and money is collected; in the transmission and reception of information and data; in transmission and reception of electrical energy in urban, commercial and industrial sectors; in the different production systems when supplies and raw materials are delivered and finished or semi-finished products are collected. The VRPSPD is also applied in cases in which new products are delivered and obsolete products or products whose expiration date has already passed are collected, with the purpose of giving them an adequate final disposal.

In practical terms, a large part of the processes in organizations where VRP is applied are developed empirically and, in general, transportation costs are high without ignoring the environmental impact and the questionable level of service to the end customer. That is why, the efforts oriented to the improvement and the search for a scientific solution of the VRP constitute another contribution of this article.

The structure of this article is made up of the following parts: 1. Introduction. 2. Description of the problem. 3.
Formulation of the mathematical model. 4. Description of the application of the Chu-Beasley genetic algorithm for the solution of VRPSPD. 5. Experimental results of the applications of the proposed heuristics formed by the Chu-Beasley genetic algorithm and exact techniques for a depot, fifty customers and four vehicles. 6. Sensitivity analysis of the results obtained with the Chu-Beasley genetic algorithm on varying the population size. 7. Conclusions and the list of bibliographical references consulted.

II. DESCRIPTION OF THE PROBLEM

It is very important to bear in mind that, in this case, the different scenarios where the deliveries and pickups of products or people that must be transported between an origin and a destination are presented are part of the VRP and the system restrictions must be complied with, including the capacity of the vehicles used. Because of the bibliographic review carried out, it was evidenced that there are three groups of VRPs as described below:

- VRP with backhauls VRPBH: in this version, deliveries are served first and then pick-ups [4].
- VRP with generation of routes using mixed vehicles MVRP: here the clients request one of the two services [8].
- VRP with simultaneous deliveries and pickups – VRPSPD: in this version customers request both services, deliver and collect products or people, guaranteeing a 100% level of service and using a vehicle on each route [25] [26] [27] [28].

For the construction of this article, the latter group was chosen, characterized by the presence of a warehouse, several vehicles and customers who request delivery and pickup services simultaneously. Therefore, the problem is solved when the set of routes that satisfy the constraints of the system is determined, which are listed below:

- Only one visit is made to each client.
- The total load transported in each vehicle must not exceed its capacity. That is, infeasibility is not allowed. In Fig. 1 you can see the situation of the VRPSPD using three vehicles and the corresponding routes:

III. FORMULATION OF THE MATHEMATICAL MODEL

Another significant aspect in the feasibility of the VRPSPD is the determination of the sequence of the chosen route, which must be viable if, when verifying the services required by the client, these do not exceed the capacity of the vehicles. For a better understanding of this version of the VRPSPD, you can see Fig. 2, where the respective feasible or infeasible routes are presented:

- Route 1: 0 → 1 → 2 → 3 → 0 feasible route, distance traveled 135-length unit.
- Route 2: 0 → 1 → 3 → 2 → 0 infeasible route, distance traveled 136-length unit.
- Route 3: 0 → 2 → 1 → 3 → 0 feasible route.
- Route 4: 0 → 2 → 3 → 1 → 0 infeasible route.
- Route 5: 0 → 3 → 1 → 2 → 0 infeasible route.
- Route 6: 0 → 3 → 2 → 1 → 0 infeasible route.

The shortest route in this case is 0 → 1 → 2 → 3 → 0, which corresponds to the optimal solution.

The mathematical model applied in this article for the solution of the VRPSPD was proposed by Dell’Amico et al [28] and used by [29]. Its description is:

\[
A = \{i, j\} \in E_k
\]
For the purpose of applying stronger inequalities for the quantities of outstanding products or goods to be collected, \( P_{ij} \), the constraint (9) can be replaced by the constraint (12) and the constraint (7) can be replaced by the inequality (13).

In the constraint (14) it is inferred that in each edge not adjacent to the warehouse merchandise is only transported once at most.

\[
Z_{min} = \min \sum_{i \in V} \sum_{j \in V} C_{ij} x_{ij} 
\]  

\[
\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V' 
\]  

\[
\sum_{j \in V} x_{ji} = 1 \quad \forall i \in V' 
\]  

\[
\sum_{j \in V} x_{ij} \leq m 
\]  

\[
\sum_{j \in V} D_{ij} - \sum_{j \in V} D_{ij} = d_i \quad \forall i \in V' 
\]  

\[
\sum_{j \in V} P_{ij} - \sum_{j \in V} P_{ji} = p_i \quad \forall i \in V' 
\]  

\[
D_i + P_i \leq Q x_i \quad \forall (i, j) \in A 
\]  

\[
D_i \geq 0 \quad \forall (i, j) \in A 
\]  

\[
x_i \in \{0,1\} \quad \forall (i, j) \in A 
\]  

\[
(Q - d_i) x_{ij} \geq D_{ij} \geq \gamma_i d_{ij} \quad \forall (i, j) \in A 
\]  

\[
(Q - P_i) x_{ij} \geq P_{ij} \geq \gamma_i p_{ij} \quad \forall (i, j) \in A 
\]  

\[
(Q - \max\{0, P_i - d_j, d_i - P_i\}) x_{ij} \geq D_{ij} + P_{ij} \quad \forall (i, j) \in A 
\]

\[
i \geq x_{ij} + x_{ji} \forall i, j, \ i < j, \ i, j \in V' 
\]

IV. DESCRIPTION OF THE APPLICATION OF THE CHU-BEASLEY GENETIC ALGORITHM FOR THE SOLUTION OF VRPSPD.

It is known by all that the application of the exact methods to solve NP-Hard type problems such as the VRPSPD is that they require a lot of computation time due to the considerable number of decision variables, system restrictions, population size and number of customers who must be served in delivery and pickup services. Therefore, the use of metaheuristics, in this case, the Chu-Beasley genetic algorithm, allows obtaining good answers, without being optimal in relatively short computation times compared to the exact methods.
It can be seen in [31] that the Chu-Beasley genetic algorithm has certain characteristics that make it more effective, such as: it uses the objective function to determine the value of the best quality solution and recognizes the infeasibility in the substitution process of a solution generated in the execution of the algorithm; replaces one individual at a time in each generation cycle; to avoid the premature tendency to optimal local solutions, each individual that joins the population must be different from all those that make up the current population; contains an aspiration criterion, even if the new individual does not meet the controlled diversity condition; it includes an improvement stage, after recombination, and based on conclusive intra-route and inter-route strategies, a feasible solution is evaluated before deciding if it can be part of the current population.

The application of the Chu-Beasley genetic algorithm contains the following stages:

A. Construction of the Initial Population

This stage is composed of: a first component that corresponds to the configurations of the routes obtained from certain constructive heuristics, and a second component is the configurations of the routes obtained in a random controlled manner. For each configuration, the objective function (fitness) and the infeasibility associated with the load of each vehicle on the routes are assessed. Fig. 3 shows a VRPSD configuration or solution for 20 nodes or clients and its coding.

```
| 20 | 6 | 2 | 8 | 13 | 3 | 9 | 15 | 16 | 1 | 17 | 10 | 19 | 12 | 4 | 11 | 18 | 5 | 14 | 7 |
```

Fig. 3. Genetic representation of a configuration of 20 nodes or customers.

B. Genetic Operators

Three genetic operators are applied in this article: selection, recombination and mutation. Following is its description:

1) Selection

In this stage operator, it is recommended to use the tournament selection method. For this, two tournaments are held where the intervention of all the individuals (customers) of the current population \( k \) is necessary. Their selection is random, then the values of their objective functions and their infeasibility are compared, if they occur, deciding on the one that has the best value of the objective function if the \( k \) individuals are feasible, or selecting the one with the least infeasibility if the \( k \) individuals are not feasible.

Since the presence of feasible and infeasible individuals is possible in this operator, the feasible individual with the best objective function is preferred. Of these two tournaments, two parents remain, which pass to the recombination stage.

2) Recombination

This operator provides the exchange of information present in the two parents, and two descendants are originated that possess genetic material from parent 1 and parent 2. Since the algorithm is elitist, only one of the children passes into the current population.

To carry out the recombination in the specialized literature there are techniques such as the “single point
crossover”, the “two-point crossover”, the “uniform crossover”, the “partially mapped crossover (PMX), the “supported crossover in order (OBX), the “cycle cross (CX)”, the “multi-parent cross (MPX)”. The “longest common subsequence crossover (LCSX)”, to name a few.

This article applies various forms of recombination that describe the PMX method, the procedure of which is:

Starting with the configuration of the parents \( P_1 \) and \( P_2 \), a common random fragment or segment is chosen and copied to the child of \( P_1 \). Starting from the first crossing point, the elements \( i \) of this segment that have not been copied are in \( P_2 \). For each of these elements \( i \), it is sought which elements \( j \) have been copied into position from \( P_1 \). Then \( i \) is put at position \( j \) taken from \( P_2 \), and it turns out that this element is not there, because it has been copied before.

If the place occupied by \( j \) in \( P_2 \) has already been filled in the child \( k \), \( i \) is installed in the place occupied by \( k \) in \( P_2 \). The other remaining elements are obtained from \( P_2 \), thus ending the procedure (See Fig. 4).

Various rates were applied to recombination, with the value 0.8 being the best results achieved.

\[ \begin{array}{cccccccccccc}
\text{P}_i & 0 & 1 & 8 & 9 & 10 & 11 & 12 & 5 & 7 & 6 & 3 & 2 & 4 & 0 \\
\text{P}_j & 0 & 10 & 9 & 6 & 5 & 7 & 8 & 12 & 11 & 4 & 1 & 2 & 3 & 0 \\
\text{H}_k & 0 & 1 & 8 & 9 & 10 & 11 & 12 & 6 & 5 & 4 & 7 & 2 & 3 & 0 \\
\end{array} \]

Fig. 4 Application of the recombination PMX in a configuration of twelve customers.

\( P_1, P_2 \) are the parents and \( H_k \) is the child who is the result of the recombination.

3) Mutation

With the application of this genetic operator some changes can be made, or parts or fragments of the solution obtained in the recombination can be altered. That is, the mutation can improve the current solution, incorporating small modifications in the process. The mutation rate used in this article is 0.05. Some strategies are known that facilitate the performance of the mutation, such as:

- Determination of local inter-route sequence using the strategies shift \((1, 0)\), shift \((2, 0)\), shift \((3, 0)\), which allow the transfer or movement of 1, 2, 3 customers from one route to another.
- Determination of inter-route local sequence using the swap \((1, 1)\), swap \((2, 1)\), swap \((2, 2)\) strategies, which make it possible to exchange 1, 2 or 3 customers between one route and another.
- Local intra-route assignment. In this situation, customer neighborhood criteria are applied to make transfers on the same route. Useful strategies in this assignment are rotation and 2-Opt. The latter has been considered as a local search algorithm that was designed by Croes in 1958 to solve the problem of a travel agency. It starts from the base of a route that crosses itself and can be reorganized, avoiding the presence of a crossing.

For the purpose of this article, some exchange strategies between customers have been used and it is illustrated with an example of swap \((1, 1)\), where the exchange of client 3 of route 3 for client 4 of route 1 is evidenced. See Fig. 5.

\[ \begin{array}{cccccccccccc}
\text{R}_1 & 0 & 6 & 7 & 5 & 4 & 9 & 10 & 11 & 12 & 8 & 1 & 2 & 3 & 0 \\
\text{R}_2 & 0 & 6 & 7 & 5 & 3 & 9 & 10 & 11 & 12 & 8 & 1 & 2 & 4 & 0 \\
\text{R}_3 & & & & & & & & & & & & & & \\
\end{array} \]

Fig. 5 Swap used \((1,1)\) in the local determination between routes.

It can be verified that when applying the swap strategy \((1, 1)\) in the mutation, another configuration is obtained with an incumbent that is guaranteed to go to the improvement stage.

C. Process of improvement of an individual

After selection, recombination and mutation are applied, the resulting configurations are subjected to a local improvement phase, which allows the separation of the individual routes and the determination or construction of subproblems of a single route each with a single deposit, with less customers than the complete configuration. The solution of this problem is achieved through the exact
branch-and-cut method, and the partial routes are established by means of the optimal solutions. The configuration obtained becomes part of the replacement stage.

D. Replacement stage

All the configurations obtained in the previous stages are compared with the individuals (customers) of the current population. A substitution of one of the individuals (customers) of the population is carried out, considering the feasibility over the infeasibility and the best objective function when feasible solutions are compared.

V. EXPERIMENTAL RESULTS OF THE APPLICATION OF THE PROPOSED METAHEURISTICS FORMED BY THE CHU-BEASLEY GENETIC ALGORITHM AND EXACT TECHNIQUES FOR A DEPOT, FIFTY CUSTOMERS AND FOUR VEHICLES.

During the bibliographic review of the literature referring to the VRPSPD, three classes of test problems have been established:

- Dethloff’s proposal: it provides 40 reference instances with 50 customers and the number of vehicles used were 4, 9 and 10 [5].
- Salhi and Nagy’s proposal: They suggest 14 instances, the number of customers is between 50 and 199 and the number of vehicles used in the process was 3, 4, 5, 6, 7 and 10 [32].
- Proposal by Montané and Galvão: they consider 12 instances. The customers are between 100-200 and the number of vehicles were 3, 5, 9, 10, 12, 16, 23 and 28 [33].

In this article, some of the Dethloff instances were used for the experimental tests, whose solutions, including the optimal limits, have been widely published in scientific literature [34] [35].

As it has been shown, the methodology used by the authors of the article is the mathematical one integrated by the Chu-Beasley genetic algorithm and the exact techniques such as mixed integer linear programming.

The results of the data processing for this experiment can be seen in Table I, with the use of the Dethloff CON 3-8 instance.

Observing table I, it is evident that, out of 24 experiments, two configurations were obtained with values of the objective function of 538.77 units of length (experiment 8) and 537.35 units of length (experiment 21), which if compared with the value 523.05 length units reported by [20] as the optimal value of the instance, are very good results in relatively short processing times (13.77 min and 14.45 min respectively) compared to the 32.05 minutes that the processing reported by [20].

Based on the configuration achieved through the Chu-Beasley genetic algorithm, whose objective function value is 537.35 distance units, the four routes traveled by the four vehicles are determined.
### Table I

**RESULTS OF DATA PROCESSING USING THE CHU-BEASLEY GENETIC ALGORITHM WITH THE DETHLOFF CON3-8 INSTANCE**

<table>
<thead>
<tr>
<th>Number of experiments</th>
<th>Population size</th>
<th>Last generation with the best objective function</th>
<th>A</th>
<th>B</th>
<th>C(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>9,546,567</td>
<td>539.27</td>
<td>539.21</td>
<td>826.57</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>169,195,901</td>
<td>634.55</td>
<td>631.46</td>
<td>6,022.12</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>14,797,375</td>
<td>606.85</td>
<td>604.4</td>
<td>621.7</td>
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<tr>
<td>4</td>
<td>200</td>
<td>18,281,903</td>
<td>604.65</td>
<td>601.58</td>
<td>865.82</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>25,751,134</td>
<td>569.14</td>
<td>566.39</td>
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<tr>
<td>6</td>
<td>200</td>
<td>3,875,937</td>
<td>620.87</td>
<td>618</td>
<td>272.65</td>
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<tr>
<td>7</td>
<td>200</td>
<td>21,615,847</td>
<td>608.97</td>
<td>606.91</td>
<td>750.12</td>
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<td>8</td>
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<td>13,634,094</td>
<td>543.37</td>
<td>538.77</td>
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<tr>
<td>9</td>
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<td>545.58</td>
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<td>101.56</td>
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<td>1,795,436</td>
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<td>610.4</td>
<td>107.83</td>
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<td>13</td>
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<td>139,028</td>
<td>620.87</td>
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<td>14</td>
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<td>15</td>
<td>200</td>
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<td>16</td>
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<td>17</td>
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<td>593.75</td>
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<td>602.19</td>
<td>851.24</td>
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<td>4,786.88</td>
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<td>24</td>
<td>200</td>
<td>50,972,210</td>
<td>573.78</td>
<td>571.4</td>
<td>1,836.31</td>
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</tbody>
</table>

A: Value best achieved by the objective function in the respective generation.
B: Worst value achieved by the objective function in the respective generation.
C: Processing time for the respective generation (in seconds).

### Table II

**SOLUTIONS ACHIEVED BY APPLYING MIXED INTEGER LINEAR PROGRAMMING**

<table>
<thead>
<tr>
<th>Vehicle /Routes</th>
<th>Continuity of established routes</th>
<th>Matheuristic solution</th>
<th>Chu - Beasley solution</th>
<th>Best possible solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle /route 1 dep</td>
<td>25 27 42 36 37 24 4 9 2 43 50 31 29 39 dep</td>
<td>194.7</td>
<td>196.63</td>
<td>190.26</td>
</tr>
<tr>
<td>Vehicle /route 2 dep</td>
<td>14 21 22 12 6 41 18 15 26 19 48 49 3 dep</td>
<td>128.7</td>
<td>126.73</td>
<td>116.52</td>
</tr>
<tr>
<td>Vehicle /route 3 dep</td>
<td>35 45 16 5 10 30 1 13 17 32 44 28 33 7 46 20 47 8 11 dep</td>
<td>183.73</td>
<td>184.62</td>
<td>183.73</td>
</tr>
<tr>
<td>Vehicle /route 4 dep</td>
<td>40 23 38 34 dep</td>
<td>31.2</td>
<td>31.2</td>
<td>31.2</td>
</tr>
</tbody>
</table>

Total distance traveled obtained by matheuristic **538.33**

Total distance traveled achieved by the Chu - Beasley genetic algorithm **539.08**

Optimal solution of the VRPSPD according to Subramanian (2012) **523.05**
Under these conditions, each of these four routes is considered as a small linear programming problem, and its solution is very easily obtained by applying the respective mathematical model. That is, the proposed heuristics is applied, which was supported by a good ordering generated by the application of the Chu-Beasley genetic algorithm, with two constructive algorithms that were designed by the authors:

- An algorithm that generates the route matrices and the quantity of products that must be delivered and collected during the visit to each customer or node of each of the routes.
- A second algorithm that controls the load on each route, avoiding infeasible configurations.

With the data from the previous algorithms, the mixed integer linear mathematical programming model is applied. The solutions obtained for the four routes can be seen in Table II.

VI. SENSIBILITY ANALYSIS OF THE RESULTS OBTAINED WITH THE CHU-BEASLEY GENETIC ALGORITHM ON VARYING THE POPULATION SIZE

It is good to remember that a sensitivity analysis can focus on different scenarios such as:

- Sensitivity analysis to assess the performance of the Chu-Beasley genetic algorithm when recombination and mutation rates are modified.
- Sensitivity analysis to assess the performance of the Chu-Beasley genetic algorithm when the size of the population is varied.
- Sensitivity analysis to assess the performance of the Chu-Beasley genetic algorithm when recombination and mutation rates are modified and the variation in population size is also included.

In this article, this analysis is made for the second option: varying the size of the population and its results can be seen in Table IV for three sizes of the population with configurations of 100, 200 and 300 clients.

Analyzing the results of the sensitivity analysis shown in Table III and the performance of the objective function for different population sizes in Fig. 6, the following aspects stand out:

For a population size of 100 clients, the objective function stabilizes with a value of 550.81 units of length between 2,500,000 and 7,500,000 generations and from 8,000,000 generations another incumbent appears with a better value of the objective function, 537.63 units of length.

For a population size of 200 clients, the objective function is fixed with a value of 537.35 units of length from 6,000,000 generations, which in any case is very close to the optimal solution reported by [23].

For a population size of 300 clients, the objective function is ensured with a value of 650.17 length units from 5,000,000 generations.

The coding of the Chu-Beasley genetic algorithm was carried out in C++ considering the following aspects:

- The configurations achieved in the process with the best objective functions.
- Each of the routes determined in the solution was treated as a subproblem that was solved by the exact method, using CPLEX Studio Optimization version 12.4.
- The process of applied heuristics was carried out on three computers with the following technical specifications as can be seen in Table IV.

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**TABLE III**

SENSITIVITY ANALYSIS FOR DIFFERENT POPULATION SIZES IN MULTIPLE GENERATIONS IN THE IMPLEMENTATION OF THE CHU – BEASLEY ALGORITHM USING THE DETHLLOFF INSTANCE WITH 3-8

<table>
<thead>
<tr>
<th>Population size</th>
<th>Generation of evolutive process</th>
<th>Objective function - OF(100)</th>
<th>Objective function - OF(200)</th>
<th>Objective function - OF(300)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>500,000</td>
<td>668.06</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>1,000,000</td>
<td>594.55</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>1,500,000</td>
<td>589.76</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>2,000,000</td>
<td>555.06</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>2,500,000</td>
<td>550.81</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>3,000,000</td>
<td>550.81</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>3,500,000</td>
<td>550.81</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>4,000,000</td>
<td>550.81</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>4,500,000</td>
<td>550.81</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>5,000,000</td>
<td>550.81</td>
</tr>
<tr>
<td>100</td>
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<td>550.81</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
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<td>550.81</td>
</tr>
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<td>6,500,000</td>
<td>550.81</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>7,000,000</td>
<td>550.81</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>7,500,000</td>
<td>550.81</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>8,000,000</td>
<td>537.63</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>8,500,000</td>
<td>537.63</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>9,000,000</td>
<td>537.63</td>
</tr>
</tbody>
</table>

---

**TABLE IV**

TECHNICAL SPECIFICATIONS OF THE COMPUTER EQUIPMENT USED

<table>
<thead>
<tr>
<th>Lenovo B40 Laptop</th>
<th>Dell Latitude E6500 Laptop</th>
<th>Lenovo Personal Computer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intel processor core (TM) 1.70 GHz- 2.40 GHz x4.</td>
<td>Intel processor core (TM) 2 Duo – 2.8 GHz – 2.80 GHz</td>
<td>Intel processor core (TM) 3.00 GHz – 3.00GHz x4</td>
</tr>
<tr>
<td>RAM memory: 4:00 GB</td>
<td>RAM memory: 4:00 GB</td>
<td>RAM memory: 8:00 GB</td>
</tr>
<tr>
<td>64 bits OS</td>
<td>GB 64 bits OS</td>
<td>GB 64 bits OS</td>
</tr>
</tbody>
</table>

---

**Fig. 6** Performance of the objective function for different population sizes
The proposed matheuristic, resulting from the integration of the Chu-Beasley genetic algorithm and the exact techniques supported by the branch-and-cut method, becomes a very good option to solve the VRPSPD for large customers or nodes, because for the same nature of NP-Hard, the mixed integer linear programming - MILP, does not solve it within a reasonable time horizon.

The incorporation of the constructive heuristic algorithms designed by the authors of the article, which facilitate the generation of the matrix of the routes from the configuration with the best incumbent, and the determination of the number of customers per route, from the control of load per vehicle, together with the matheuristic described in this article, show that very good results are obtained in relatively short processing times, ratified by the sensitivity analysis carried out for different sizes of the customer population.

The scope of the proposed matheuristic to solve this class of routing problems is promising and is conditioned to the knowledge and skills of the researchers, to achieve an excellent execution, making the corresponding adjustments and ensure a good performance in the simulations without ignoring that the matheuristics do not guarantee the global optimal solution of routing problems.

On the other hand, the challenge arises of integrating the environmental effects into this situation and carrying out the respective tests, exploring the instances available in other works published in indexed journals or creating the instances themselves.

REFERENCES

Pedro Pablo Ballesteros Silva. Industrial Engineer, Production Engineering Specialist of the Francisco José de Caldas District University of Bogotá; Master in Operations Research and Statistics, PhD. in Engineering from the Technological University of Pereira.

Pedro Pablo has worked in many private and official companies: Colcarga, Temecal S.A., Industria Militar, Colombian Institute for the Promotion of Higher Education - ICFES. He has worked as a university professor at the Francisco José de Caldas District University, Cundinamarca University, Los Libertadores University Foundation and Technological University of Pereira. In this university he has been titular professor and has guided the subjects Times and Methods, Linear Programming, Operations Management I and II, Logistics . He has authored more than fifty scientific articles in the areas of Logistics, administration of Operations, Simulation and others. He was the creator and coordinator of the Postgraduate Program Specialization in Business Logistics. He has been a visiting professor at several universities abroad such as Universidad Nacional del Sur in Bahía Blanca (Argentina) where he directed the course The logistics from a point of business view at the Doctorate of Engineering; at the Technological University of San Juan del Río in the state of QUERETARO (Mexico) where he directed the course “The importance of reverse logistics in the rescue of the environment”. He has been an international speaker at events such as EPIO-ENDIO, CLAIO, Ph. D. Ballesteros Silva obtained from the Ministry of Interior of Colombia in 2018 the Certificate of Logic Support Registry - Software awarded for the work entitled “Application of Matheuristic Techniques for the Solution of the Vehicle Routing Problem with Simultaneous Deliveries and Pickups - VRSPDP”

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Diana Paola Ballesteros Riveros. I am a professional with a global vision and research mentality. During 8 years of experience as Industrial Engineering, University Professor and Master in Economic and Financial Administration, I have demonstrated my ability to generate and implement initiatives to solve problems in public and private organizations, with a firm commitment to sustainable development, and a high sense of ownership and responsibility.

Diana Paola has worked in the following companies: Banco Interamericano de Desarrollo - BID, Confecámaras, Chamber of Commerce of Pereira and Gobernación de Risaralda as Financial Advisor- Empréndelo Project; Audifarma S.A. - Supply Chain Management as Supply Chain Analyst; Technological University of Pereira - Unit of Social and Welfare University as University Professional and, University Professor. He has authored more than twenty scientific articles in the areas of Logistics, administration of Operations,
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