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Learning of platonic solids by using augmented reality and theory of didactic situations

Aprendizaje de los sólidos platónicos mediante el uso de la realidad aumentada y la teoría de las situaciones didácticas

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***Abstract*—Augmented reality (AR) is a technology that combines in real time environment information with the lived information a virtual environment. This technology can be used together with the theory of didactic situations (TDS) for the learning of spatial geometry. The aim of this study was to evaluate the effect of an intervention using AR and TDS on the learning of Platonic solids (PS) in junior high school (7th degree) students. 34 students were allotted in both an experimental group (n=17) and a control group (n=17). Students in the experimental group received a didactic sequence related to PS using AR and those in the control Group received a traditional study class with a 3D manipulatives. The learning of students in both groups was assessed with a semiquantitative scale (from 1 to 4) comparing data gathered at the beginning of the study (pretest) with data obtained after applying the didactic sequence or receiving the traditional study class (posttest). Pretest data revealed that the starting knowledge about PS was similar between both groups. Posttest data showed that students of the experimental group learned better about PS than students of the control Group. The use of AR and TDS could improve the learning PS in students because they can easily identify and interact with the common patterns of this geometric elements as if these were real objects.**

***Index Terms*— Augmented Reality, Geometry, Learning, Platonic solids, Theory of Didactic situation.**

***Resumen*— La realidad aumentada (RA) es una tecnología que combina información del entorno en tiempo real con la información vivida en un entorno virtual. Esta tecnología se puede utilizar junto con la teoría de situaciones didácticas (TSD) para el aprendizaje de la geometría espacial. El objetivo de este estudio fue evaluar el efecto de una intervención usando RA y la TSD en el aprendizaje de los sólidos platónicos (SP) en estudiantes de secundaria (7º grado). Se asignaron 34 estudiantes tanto en un grupo experimental (n=17) como en un grupo de control (n=17). Los estudiantes del grupo experimental recibieron una secuencia didáctica relacionada con los SP usando RA y los del grupo control recibieron una clase de estudio tradicional con manipulativos en 3D. El aprendizaje de los estudiantes de ambos grupos se evaluó con una escala semicuantitativa (del 1 al 4) comparando los datos recogidos al inicio del estudio (pretest) con los obtenidos después de aplicar la secuencia didáctica o recibir la clase de estudio tradicional (postest). Los datos de la prueba preliminar revelaron que el conocimiento inicial sobre los SP era similar entre ambos grupos. Los datos posteriores a la prueba mostraron que los estudiantes del grupo experimental aprendieron mejor sobre PS que los estudiantes del grupo de control. El uso de AR y TDS podría mejorar el aprendizaje los SP en los estudiantes porque pueden identificar e interactuar fácilmente con los patrones comunes de estos elementos geométricos como si estos fueran objetos reales.**

***Palabras claves*—Aprendizaje, Geometría, Realidad Aumentada, Sólidos Platónicos, Teoría de las situaciones didácticas.**

I. INTRODUCTION

AUGMENTED reality (AR) is an emerging technology that combines the objects of both the real and the virtual world [1], allowing the interaction in real time of the information of the natural environment with the information that is lived in the virtual environment [2]. Thus, the user can employ the senses without leaving the real world. The implementation of AR for teaching mathematics; particularly, in the field of geometry in junior high school education has gained relevance because it enables the study of three-dimensional models and their two-dimensional representations from different points of view [3]. This allows the students to understand the concepts of geometric representation and their interaction in real time so that the learner can discover the characteristics and fundamental properties of three-dimension (3D) objects [4].

Several studies [1]; [5]; [6]; [7]; [8], have suggested that AR favors the student motivation for learning mathematics, making this technology a valuable resource for teachers at the time of teaching and that can be applied in any area or level of training due to its versatility.

On the other hand, several authors have carried out works on the importance of AR in the learning and teaching of geometry, because AR favors the acquisition of abilities for spatial visualization by favoring the retention of the memory of the objects to be studied over long term [9]; [10].

Although AR is a technological mediator, this cannot be considered as relevant if it is not accompanied by a didactic

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mediator, such as the theory of didactic situations (TDS) used in this study [11]. In didactic situations the student learns through the adaptation to a learning environment that is a factor of contradictions [12], difficulties, and imbalances, a little like human society does, this knowledge is the result of the adaptation of the student, which manifests itself through new responses that are the mark of learning [13]. According to the TDS, the selected learning environment (in this case AR) must be designed in such a way that the knowledge resulting from adaptation learning is similar to the knowledge that you want to teach without the need to be explained by the teacher [14]

The synergy between TDS and AR can be considered as a valuable element within the mathematics didactics that should be taken into account by teachers at all education levels because it facilitates the understanding of abstract concepts through the manipulation of 3D objects with sequences of learning by adaptation, which represent an invaluable couple for the teaching-learning in geometry [1]; [15]

This research aimed to compare the effect produced by AR and TDS on the learning of PS in seventh-grade high school students. We hypothesized that the implementation of a didactic sequence with AR mediated by TDS favors the learning of the concepts of the PS in students.

II. Methodology

This study was approved by the local ethical committee on education of the Master Program in Didactics of Mathematics of the Universidad de Caldas. It was conducted for six months and included two experimental groups of 7th-grade students in a public urban high school from Manizales, Caldas, Colombia.

A. Study design

The approach of the present research was quantitative with a descriptive and correlational scop, from which two hypotheses were formulated (null and alternative) about the possible relationships between the two variables that were included in the problem of the study. Further, the study was considered as quasi-experimental with two intact groups (experimental (EG) and control groups (CG)) from 7th degree from the similar high school education with similar characteristics and particularities (i.e.: age, economic stratum, place, parents' education, amongst others). To note, all the students attended the mathematics class with the same teacher.

The EG was composed of 17 students who were taught with the use of an AR App (AR Platonic Solids (Spanish Version 1.0)) designated by us using the academic license by Unity Technologies, which is a Danish-American video-game software development company based in San Francisco, CA, USA [16], and a didactic sequence called PS Learning using AR designed with the didactic approach of the TDS that was composed by four didactic situations: action, formulation, validation, and institutionalizing.

Briefly, during the first 45 min the students received the TDS guide, were trained in the management of the AR App, and performed the first didactic situation (action), which consisted in solving the question (cognitive conflict): “Is it possible to made convex polyhedrons with regular polygons?”. After a break of 10 min, the second lapse of 45 min was used for performing the second didactic situation (formulation), in which the students came together to reach an agreement about the question of the first didactic moment. A second 10 min break was allowed to continue with a 45 min session in which was performed the third didactic situation (validation) by using mathematics demonstration by Euler`s theorem. Furthermore, a 45 min session was used for performing the four didactic situations (institutionalizing), in which the students made an oral explanation of their findings and the teacher clarify inquietudes, mistakes, and concepts and finally made a final conclusion for institutionalizing the concept about what a PS is.

The control group also included 17 students, who did not receive the intervention but were taught with the same theory through the master class and the use of manipulatives. Two independent master classes with 90 min of duration were performed, and additional homework for an independent task for the students with a mean time of 90 min was recommended. An entrance test (pretest) and an exit test (posttest) were applied to both groups. In line with this, a rubric (Table I) was designed for evaluating the gained learning of the students in relation to a geometry knowledge exam that included pre and post-test questions with both dichotomic natures (yes or no type) or of free answering (open).

TABLE I
RUBRIC FOR ASSESSING THE ADVANCE OF LEARNING OF PLATONIC SOLIDS IN TWO GROUPS OF 7TH HIGH SCHOOL STUDENTS

Degree of learning	Score	Specific criteria
Low	1	From 0 to 5 right answers in the questionary used for both pre and post-test.
Basic	2	From 6 to 10 right answers in the questionary used for both pre and post-test.
Advanced	3	From 11 to 15 right answers in the questionary used for both pre and post-test.
Superior	4	From 16 to 20 right answers in the questionary used for both pre and post-test.

A summary of the design of the study is shown in Fig. 1.

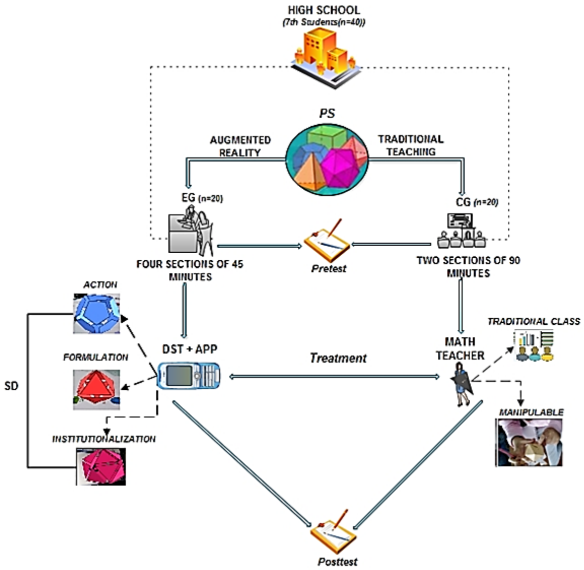

Fig. 1. Schematic representation of the study`s design.

B. Statistical analysis

Initially, data was evaluated for normality by a Shapiro-Wilk test, after this test data exhibited a non-parametric distribution ($P < 0.05$), reason why the medians of the groups were compared by using a non-parametric ANOVA (Kruskal-Wallis test), followed, when necessary, by either non-paired and paired non-parametric comparisons (i.e.: Wilcoxon and U Mann-Whitney test. A $P < 0.05$ was accepted as statistically significant for all the tests.

III. RESULTS

All 34 participants could successfully finish this study. In general, students from both groups were receptive and motivated during the experiment, but the students from the EG group were apparently more enthusiastic and demonstrated interest in learning through the use of AR plus TDS.

From the statistic point of view the nature of the data gathered in the present study was non-parametric, such as presented in table II after performing a Shapiro-Wilk test.

TABLE II
SHAPIRO-WILK TEST FOR EVALUATED MEDIAN GROUPS AT PRE- AND POST-TEST

Tests of Normality							
		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Group	Statistic	df	Sig.	Statistic	df	Sig.
Var_Rpta	CGPr	0.497	17	0.000	0.470	17	0.000
	EGPr	0.440	17	0.000	0.579	17	0.000
	CGPo	0.469	17	0.000	0.533	17	0.000
	EGPo	0.300	17	0.000	0.752	17	0.000

a. Lilliefors Significance Correction

Then we compared the medians from the 4 evaluated groups by using a Kruskal-Wallis test in order to diminish the probability of accept or reject the null hypothesis when this was true or false (type I or III errors). This a-priori test is fundamental before comparing the medians by non-parametric paired tests (Table II).

TABLE III
RESULTS FOR KRUSKAL-WALLIS TEST PERFORMED TO ALL MEDIAN GROUPS AT THE PRE- AND POST-TEST MOMENTS.

Test Statistics [a,b]	
	Var_Rpta
Kruskal-Wallis H	38,119
df	3
Asymp. Sig.	0.000

a. Kruskal Wallis Test
b. Grouping Variable: Group_PrPo

Finally, non-paired and paired non-parametric tests were run to compare the potential differences between the medians from the groups. In table III appears the results from group medians compared at the pre-test moment, which demonstrated that both medians were statistically similar.

TABLE IV
RESULTS FOR U-MANN-WHITNEY TEST PERFORMED TO THE MEDIAN GROUPS AT THE PRE-TEST MOMENT.

Estadísticos de prueba [a,b]	
	Var_Rpta
Mann-Whitney U	127,500
Wilcoxon W	280,500
Z	-0.797
Asymp. Sig. (2-tailed)	0.426
Exact Sig. [2*(1-tailed Sig.)]. unilateral)]	0.563[b]

a. Grouping Variable: Group_PrPo
b. Not corrected for ties.

In table IV are show the output data from the paired comparisons form both group medians at the post-test moment. At this point, we observed and significant difference between both groups.

TABLE V
RESULTS FOR U-MANN-WHITNEY TEST PERFORMED TO THE MEDIAN GROUPS AT THE POS-TEST MOMENT.

Test Statistics[a]	
	Var_Rpta
Mann-Whitney U	52,000
Wilcoxon W	205,000
Z	-3,678
Asymp. Sig. (2-tailed)	0.000
Exact Sig. [2*(1-tailed Sig.)].	0.001

a. Grouping Variable: Group_PrPo
b. Not corrected for ties.

Table V shows the results of the non-parametric test ($p=0.001$).

Regarding the statistical analysis from gathered data in this study, we observed that the entrance knowledge about spatial geometry, and particularly that related to PS (as measured by the pretest) was similar between both student groups. Once the treatments, either magistral class plus manipulative objects or AR plus TDS, were applied to the student groups, we noticed that those students from the EG shown significantly (P= 0.001) better rubric posttest score than the students from the control group (Fig. 2).

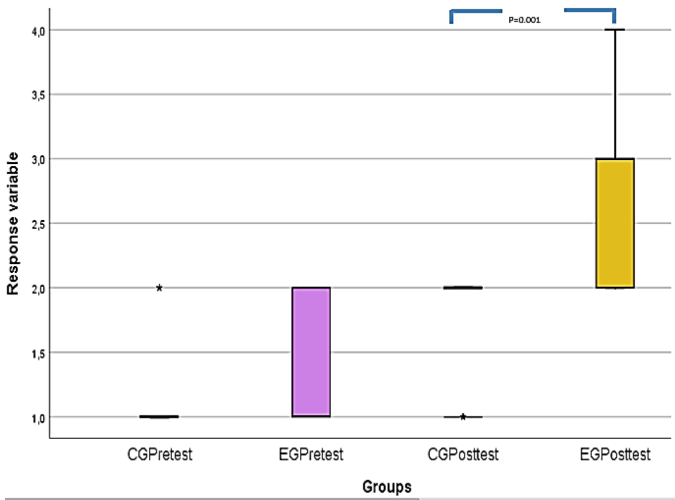

Fig. 2. Assessment of the degree of learning according to the rubric scale in the experimental (EG) and control groups (CG) during both pre and post-test interventions

IV. DISCUSSION

AR is an innovative technology that can complement the real world environ-ment with a smartphone to generate sensory inputs [15]. These virtual components seem to coexist with the real one in the same spaces to improve the perception of reality by students and enrich the information to be disclosed [17].

On the other hand, a didactic situation is a set of relationships explicitly or implicitly established between a student or a group of students, some environment (including instruments or materials, in this case, AR), and the teacher in order to allow students to learn some knowledge [11]. The union of both strengthens the teaching-learning process of mathematics, giving the student the possibility to explore and learn naturally, while the teacher designs the activities so that the student develops competencies and recreates learning from a novel didactic perspective.

The present study compared the traditional didactic approach to teach PS with a didactic method that combined TDS plus AR. The results of our research showed that this last didactic method was suitable and better than the traditional classroom approach for the teaching of the PS because the degree of learning was significantly higher in the students of the EG.

We performed an exhaustive bibliographic search focused on the use of AR and didactic mediators such as TSD. However, according our searching results, we only could find a paper about these topics, which is a recent study by our research team that evaluated the AR and the didactics engineering as a didactic mediator [18]. In general, the results obtained in the mentioned study showed that the students significantly learned the concepts of quadric surfaces thanks to the use of AR, as well as the didactic sequence mediated with the guide based on didactic engineering. This could be a very important indicator for researchers in this field, because although AR favors visualization it is necessary to guide it with a mediator or didactic approach [18].

Currently, the use of smartphones by students has become a problem for teachers because many students lose attention during traditional class and prefer using these devices to play videogames, see musical videos, consult social networks, and so forth. Thus, the teachers should change their traditional didactic methods including the use of smartphones as didactic tools and no as distraction elements during the teaching and learning process.

The present study demonstrates that the inclusion of smartphones in the teaching-learning process of PS through a TDS plus AR method could be an important approach for the teaching of geometric concepts and spatial learning and consequently favor the learning of the students in a friendly environment without the apparent deprivation of their smartphones[16];[18]; [19].

Notably, we observed that some students from the control group were anxious about using their smartphones during class. In many instances was necessary to ask some of them that did not use their smartphones during the class of PS to avoid some distraction that could produce adverse effects during their learning process and disturbing the learning process of those students that were interested in this classic approach of PS teaching. At this point, it is possible to consider that, in some extension, the significantly lower degree of learning of the students of the control group (as observed in the post-test) when compared to the students of the EG was also due to the distraction produced by the smartphones of those students that showed less interest during the class.

Traditionally, the most complicated in teaching geometric; particularly in the teaching solids, is that these figures need a 3D representation, thus some teachers are facing the problem of representing this type of objects in a traditional board by using 2D geometric solid plots, and in many situations, these plots do not represent the exact form of the 3D object (solid). The use of manipulable (3D) solids is a common methodological approach but this kind of didactic tool does not let to dissect and study the entire structure, something different happens with the use of the AR [18]. Maybe, the use of smartphones improved the learning of PS in the students of the EG.

This study had several limitations. First, the smartphones used in this study (owned by the students) were obsolete. Second, our results do not allow to distinguish if the improvements of learning were due to AR or TDS. Third, another limitation could be the small sample of participants enrolled in the study, which could prevent the generation of robust conclusions. Finally, our AR App only can be played on smartphones with Android platforms Oreo 8.1 or successive versions. It is also important that the smartphones used for this study had memory storage of 45 MB or more.

V. CONCLUSION

We concluded that those groups of students that were trained with the DST guide and instructed AR App showed significantly better comprehension of the PS when compared with the control group. Furthermore, from a subjective point of view, we appreciated that the students from the EG presented a more enthusiastic and motivated behavior in the learning of PS in comparison to the students from the control group.

However, the feasibility of using this approach depends on students' access to Android devices of given minimum specifications, which are not always guaranteed in contexts such as Latin America.

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